Galaxy Dynamics Predictions in the Nonsymmetric Gravitational Theory

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Abstract

In the weak field approximation, the nonsymmetric gravitational theory has, in addition to the Newtonian gravitational potential, a Yukawa potential produced by the exchange of a spin 1^+ boson between fermions. If the range r_0 is of order 30 kpc, then the potential due to the interaction of known neutrinos in the halos of galaxies can explain the flat rotation curves of galaxies. The results are based on a physical linear approximation to the NGT field equations and they are consistent with equivalence principle observations, other solar system gravitational experiments and the binary pulsar data.

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After two decades there has not been any observation of exotic dark matter candidates. Recent observational results using the HST have excluded faint stars as a source of dark matter in the solar neighborhood[1]. However, the galaxy dynamics observations continue to pose a serious challenge to gravitational theories. The data are in sharp contradiction with Newtonian dynamics, for virtually all spiral galaxies have rotational velocity curves which tend towards a constant value[2]. Similar results are observed in gravitational lensing[3].

As in the case of anomaly problems in solar dynamics of the past century, concerning Uranus and Mercury, there are two ways to circumvent the problem. The most popular is to postulate the existence of dark matter[4]. It is assumed that dark matter exists in massive almost spherical halos surrounding galaxies. About 90% of the mass is in the form of dark matter and this can explain the flat rotational velocity curves of galaxies. However, the scheme is not economical, because it requires two or three parameters to describe different kinds of galactic systems and no satisfactory model of galactic halos is known.

The other possible explanation for the galactic observations is to say that Newtonian gravity is not valid at galactic scales. This has been the subject of much discussion in recent years[5–9]. We know that Einstein's gravitational theory (EGT) correctly describes solar system observations and the observations of the binary pulsar PSR 1913+16[10]. Therefore, any explanation of galactic dynamics based on gravity must be contained in a modified gravitational theory that is consistent with EGT. The following constraints on a classical gravitational theory are:

(1) The theory must be generally covariant i.e., the field equations should be independent of general coordinate transformations and should reduce to special relativity dynamics in flat Minkowskian spacetime.

(2) The theory should be derivable from a least action principle in order to

guarantee the consistency of the theory.

(3) The linear approximation should be consistent i.e., there should not be any ghost poles, tachyons or higher-order poles and the asymptotic flat space boundary conditions should be satisfied.

(4) The equations of motion of test particles should be consistent with local equivalence principle tests.

(5) All solar system tests of gravity and the observed rate of decay of the binary pulsar should be predicted by the theory.

We shall now consider the predictions of a new version of the nonsymmetric gravitational theory which can satisfy all the above criteria[11−15]. The theory has a linear approximation free of ghost poles, tachyons and higher-order poles with field equations for a massive spin 1^+ boson with a range parameter, $\mu^{-1} = r_0$, corresponding to Proca-type equations for an antisymmetric potential. The expansion of the field equations about an arbitrary EGT background metric is also consistent and satisfies the physical boundary conditions at asymptotically flat infinity.

An important result of the theory is that the field equations have a spherically symmetric static solution, which is completely regular everywhere in spacetime and possesses no black hole event horizons[16,17]. Black holes are replaced by superdense objects (SDO's) which do not have null surfaces.

A derivation of the equations of motion of test particles yields the following potential in the weak field approximation[14]:

$$V(r) = -\frac{G_\infty M}{r}\left(1 - \frac{g^2}{G_\infty m_t M} Y_t Y_s e^{-\mu r}\right), \tag{1}$$

where g^2 is a coupling constant measuring the strength of the coupling of the antisymmetric field $g_{[\mu\nu]}$ to matter and Y_t and Y_s denote the NGT charges of the test particle and the

source, respectively. Moreover, G_∞ is the gravitational constant at large distances. For $r << r_0$, the Newtonian laws apply with the local gravitational constant:

$$G_0 = G_\infty(1 - \frac{g^2}{G_\infty m_t M} Y_t Y_s). \tag{2}$$

Since the exchanged boson is a spin 1^+ particle, the Yukawa potential term corresponds to a repulsive force.

We shall assume that the NGT charge is associated with fermion particles:

$$Y = Y_B + Y_\nu, \tag{3}$$

where Y_B and Y_ν denote the baryon charge and neutrino charge, respectively. Let us assume that the mass M is dominated by the rest mass of the constituents:

$$M \approx m_N(N + Z) + m_\nu N_\nu, \tag{4}$$

and that, in addition, we have

$$M \approx m_N(N + Z), \tag{5}$$

where m_N, m_ν and N_ν denote the nucleon mass, the neutrino mass and the number of neutrinos, respectively. Also, N and Z denote the number of neutrons and protons, respectively. Then, Eq.(1) can be written as

$$V(r) = -\frac{G_\infty M}{r}\left[1 - \eta_t Y_t \left(\frac{Y_s}{N + Z}\right) e^{-\mu r}\right], \tag{6}$$

where $\eta_t = g^2/G_\infty m_t m_N$.

The rotational velocity is determined by

$$v_c = \left(\frac{G_\infty M}{r}\right)^{1/2}\left[1 - (1 + \frac{r}{r_0})\eta_t Y_t \left(\frac{Y_s}{N + Z}\right) e^{-r/r_0}\right]^{1/2}. \tag{7}$$

In terms of the neutron excess, we can define

$$\rho = \frac{N-Z}{N+Z}, \tag{8}$$

and we have[18]

$$\frac{Y_B}{N+Z} = \frac{1}{\sqrt{2}}\Big[\cos(\theta - \frac{\pi}{4}) - \rho\sin(\theta - \frac{\pi}{4})\Big]. \tag{9}$$

The best observational limits on violations of the equivalence principle come from Eötvös-type experiments[19,20] that measure the differential acceleration of two bodies towards the Earth. These limit $(\delta a/g)_\oplus$, where $\delta a = a_1 - a_2$ (a_i is the measured acceleration of body i) and g is the gravitational acceleration at the Earth's surface. We have for $r_\oplus << r_0$:

$$\left(\frac{\delta a}{g}\right)_\oplus \approx \eta_B \delta Y_i \left(\frac{Y}{N+Z}\right)_\oplus < 10^{-12}, \tag{10}$$

where $\eta_B = g^2/G_\infty m_N^2$. For the two examples, $\theta = \pm\pi/4$ for which $Y_{B\pm} = (N \pm Z)/\sqrt{2}$, we have that $Y/(N+Z) \approx 10^{-2}$ for the Earth and δY_i ranges from 0 to 10^{-1} for materials compared in the experiments for Y_{B-}, while for Y_{B+} we have that $Y/(N+Z) \approx 1$ for the Earth and $\delta Y_i \sim 10^{-3}$ for differences in materials. Therefore, we conclude that $\eta_B < 10^{-9}$ in order not to violate these accurate experiments. Other constraints coming from perihelion-shift measurements and those from the binary pulsar and classical binary star systems such as DI Herculis would also be consistent with the latter bound on η_B[21].

Sanders[22,23] has done an extensive phenomenological analysis of fits to the rotational velocity curves of galaxies, using a repulsive Yukawa potential added to the attractive Newtonian potential:

$$V(r) = -\frac{G_\infty M}{r}\left(1 - \alpha e^{-r/r_0}\right), \tag{11}$$

where $\mu^{-1} = r_0 \approx 30$ kpc and $\alpha = 0.92$. However, we see that if we assume only a coupling, in NGT, to baryons, then such fits would overwhelmingly violate the equivalence principle

measurements, as noted by Sanders[6]. We shall instead assume that the dominant coupling is due to neutrino pairs with the potential:

$$V_G(r) = -\frac{G_\infty M}{r}\left(1 - \gamma_\nu e^{-r/r_0}\right), \tag{12}$$

where

$$\gamma_\nu = \frac{g^2 N_\nu}{G_\infty m_\nu m_N}\left(\frac{Y}{N+Z}\right)_G. \tag{13}$$

The equivalence principle tests and other observational gravitational tests no longer limit $\alpha = \gamma_\nu$ to small values. We can have $\gamma_\nu \sim 0.92$ or larger values of γ_ν, depending on the values of the constants m_ν and N_ν and assuming a universal value for the NGT fermion coupling constant g^2.

The present upper bound on m_ν for the electron-dominated family is 9 ev and the lower bound still includes zero[24]. The present mean number density of neutrinos plus their partners in one family, determined by the density of relict neutrinos, is fixed by the cosmic background temperature to be[25]:

$$n_\nu = 113 \text{ neutrinos}\,\text{cm}^{-3}. \tag{14}$$

A value for the neutrino mass can be obtained from estimates of the density of neutrinos in galaxy halos[25]:

$$m_\nu = \frac{70}{[r_{1/2}(\text{kpc})]^{1/2}}\left(\frac{200\,\text{km}\,s^{-1}}{v_c}\right)^{1/4}, \tag{15}$$

where $r_{1/2}$ denotes the galaxy core radius. For typical dark halos of giant galaxies, with $v_c \sim 200\,\text{km}\,s^{-1}$ and core radii $r_{1/2}$ of a few kiloparsecs, the neutrino mass obtained from (15) is similar to the mass at which neutrinos close the universe at an acceptable value of the Hubble constant. However, there are serious problems with this scenario for dwarf

spheroidal galaxies in the halo of the Milky Way[25,26]. For Draco and Ursa Minor, the resulting neutrino mass is

$$m_\nu \sim 400\, ev, \tag{16}$$

which is an order of magnitude or more above what is allowed by the mean mass density (14).

Our fits to the galaxy rotational velocity curves are not restricted by the problems with dwarf galaxies and we can obtain fits for $m_\nu \leq 1$ ev for reasonable values of N_ν. We note that $m_\nu \geq 10^9 m_N$ which means that we gain a factor of 10^9 or more in Eq.(13), when compared to the baryon coupling contribution.

Since N_ν can vary from galaxy to galaxy, we should be able to fit the observed fact that for low luminosity-low rotation velocity galaxies the rotation curve still tends to be rising at the last measured points, whereas in high rotation velocity galaxies the opposite seems to be true [6], i.e., the rotation curves are decreasing but still more slowly than is expected from the light distributions. In the empirical work of Sanders, based on Eq.(11), it was predicted that larger galaxies should exhibit larger mass discrepancies, which does not seem necessarily to follow from the data, e.g., for the very large galaxy UGC 2885, there is no evidence of any mass discrepancy out to a radius of 60 or 70 kpc. In contrast, there are very small galaxies such as the spiral UGC 2258, which display a significant mass discrepancy at a radius less than 10 kpc. Again this kind of behavior depends upon the composition of the neutrino halo of the galaxy, and a fit to the data should be possible, although further data fitting is necessary to confirm this scenario.

Regarding the dark matter problem at cosmological scales a cosmological constant Λ is one way to make a low-density universe consistent with the condition from inflation that space curvature is negligibly small, without invoking the hypothesis of exotic dark matter. A positive Λ tends to pull clusters apart, but the effect can be ignored for interesting values

of Λ. Perhaps, the increased value of G_∞, obtained in the present scheme for very large distances, could improve the clustering effect of large scale gas clouds and help to account for the formation of galaxies.

From the predicted weak field gravitational potential of NGT and the work of Sanders, we have seen that it is possible to fit a wide class of galaxy rotational velocity curves for fixed values of g^2 and μ and from variable light distributions and galaxy halo neutrino density distributions. Values of m_ν are allowed that do not contradict the experimental bounds on this constant and that are not inconsistent with cosmological estimates of the mean density of neutrinos. A positive feature of this scheme is that only the *known* neutrino dark matter needs to be postulated to fit the data. Moreover, the theory is generally covariant, posseses a physically consistent linear approximation and allows a relativistic calculation of the bending of light which agrees with the solar experiments.

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